

Complex numbers and their components

1. Mathematics is used to model the real world. There is no such thing as “10 apples”, for each apple is different, and given ten apples, one may claim there are ten, and another may claim there are nine together with one rotten apple. What defines one apple as an apple and another as a rotten apple is subjective. One could say that there are 10 uranium nuclei, for they are all identical, but suppose one spontaneously decays. At which point do the ten uranium nuclei turn into nine together with other “stuff”? However, integers are generally more than sufficient at counting items, especially when dealing in a commercial transaction, for example.

Similarly, complex numbers can be used to model the *impedance* of a circuit consisting of resistors, inductors and capacitors to an alternating current that has an angular frequency $\omega = 2\pi f$ where f is the frequency in hertz. In North America, the current is 60 Hz, so the angular frequency is $\omega = 120\pi \approx 376.99111843$. We will approximate this with $\omega = 377$.

The impedance of a resistor of R ohms is R . The impedance of capacitor with C farads is $-\frac{1}{\omega C}j$. The impedance of an inductor with L henrys is ωLj .

Now, if the impedance of a circuit is $z = \alpha + \beta j$, then it is necessary that $\alpha \geq 0$ and the real component is the resistance of a resistor. If $\beta > 0$, we have a resistor and inductor connected together in series, and the imaginary component equals ωL . If $\beta < 0$, we have a resistor and capacitor connected together in series, and the imaginary component equals $-\frac{1}{\omega C}$. Now, answer the following two questions:

- Assume we are using 60 Hz and approximating $\omega = 377$ and the impedance of a circuit is $3 + 4j$. How many ohms is the resistor? Are we connected to a capacitor or an inductor? If we are connected to a capacitor, how many farads is that capacitor? If we are connected to an inductor, how many henrys is that inductor? Find your answers to four significant digits.
- Repeat Question 1a but with an impedance of $2 - 7j$.

See https://en.wikipedia.org/wiki/Series_and_parallel_circuits#Series_circuits for information about circuits in series.

Note that in this class, you need not understand the details of why circuit elements can be modeled in this manner. You will learn this in your course on linear circuits.

Not required, but perhaps of interest. How does the above answers change if you were in Europe, where the alternating current is at 50 Hz, not 60 Hz. Consequently, $\omega = 100\pi \approx 314.1592654$. What would be the farads or henrys of a capacitor or inductor if we knew the total impedance was $3 + 4j$ or $2 - 7j$?

The justification for having different frequencies includes the desire for a monopoly over electrical equipment. The reason both are over 40 Hz is that at 40 Hz, incandescent light bulbs have been known to appear to flicker (between peaks, the tungsten cools to the point of giving off noticeably less light). In Japan, west of and including Kyoto use 60 Hz, while east and north of Tokyo uses 50 Hz.

See <https://www.djtelectricaltraining.co.uk/downloads/50Hz-Frequency.pdf> for more details.